New Look at the Large Numbers

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A new interpretation for the large number hypothesis is given, referring to the close connection between the Bekenstein-Hawking entropy and Weizsäckers ur theory.

1. INTRODUCTION

Eddington (1931, 1936) points out that the empirical values of dimensionless numbers appearing in physics may crave and admit a theoretical explanation. Two of his examples, the inverse fine structure constant $hc/e^2 \approx 137$ and the mass ratio of proton and electron, $m_p/m_e \approx 1840$, are today subsumed among the unsolved problems of elementary particle physics. Two others, known as the „large numbers," belong to gravitation and cosmology. They are

$$E_1 = e^2/Gm_em_p = F_{Coul}/F_{grav}$$

the relation between the Coulomb and gravitational forces in the hydrogen atom, and

$$E_2 = \text{number of nucleons in the universe}$$

Empirically, E_1 is of the order 10^{40} and E_2 of the order 10^{80} (if we are permitted to use a closed world model); hence

$$E_2 = E_1^2 \qquad (1)$$

Dirac (1937, 1938) proposes an explanation of the two large numbers in terms of a third one, the age of the universe t_U, the epoch, measured in units of a typical atomic time e^2/mc^3. Using the present age of the universe, it turns out that

$$E_3 \overset{\text{def}}{=} \frac{t_U}{e^2/mc^3} \approx E_1 \qquad (2)$$

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Dirac's idea is that this relation, too, should hold as a law of nature: „Any two of the large dimensionless numbers occurring in nature are connected by a simple mathematical relation, in which the coefficients are of order of magnitude unity." From (2) and (1) it would follow that E_1 and E_2 are dependent on the epoch. Hence, Dirac postulates, the number of nucleons in the universe must increase with t_U^2, and the gravitational „constant" G must decrease with t_U^{-1}. Dirac also mentions the possibility that $hc/2\pi e^2$ and/or m_p/m_e might vary proportionally to the logarithm of t_U.

The theory of a variable G is further extended by Jordan and his co-workers (Jordan, 1955; Jordan er *al,* 1964; Ludwig, 1951). A review of all theories with variable G is given by Wesson (1980). In these theories, G is taken as the only natural „constant" varying in time.

Dirac's approach can be said to suffer from two weaknesses, a theoretical and an empirical one.

Theoretically, in our present understanding of physical theory there is no logical necessity for an explanation of the large numbers by some law of nature. They might describe contingent initial conditions of the universe or of coupling constants. The striking numerical relations (1) and (2) might either be chance coincidences or they might even indicate those values of the constants at which there is a possibility of there being living organisms in the world able to observe it [see, eg., Misner *et al.* (1973)].

Empirically, no sign for a variation of G with t_U has so far been found. Hellings *et al.* (1983), analyzing the Viking lander data, have shown strong, while not yet compelling, empirical arguments against it.

In this paper we shall, however, propose a further study of the theoretical meaning of the large numbers. In the present observational situation it seems likely that a final empirical test may presuppose a better understanding of the theoretical foundation and implications of Dirac's hypothesis.

2. BLACK HOLES

The theory of black holes has given rise to a further type of large number, described by Bekenstein (1973, 1981) and Hawking (1975) as black hole entropy. For simplicity of expression we shall describe the theory in Planck-Wheeler units. For Dirac, a natural system of units consisted of *c, h,* and the mass of some elementary particle. In a black hole, elementary particles are no recognisable; hence G is then a more natural unit. Planck (1899) found that *h, c,* G, and the Boltzmann constant *k* constitute a system of natural units which „holds as long as the laws of gravitation, of propagation of light in vacuum, and the principles of thermodynamics remain valid." Our units of time, length and mass are of the order

$$t_0 = 10^{-43}\ \text{sec} \qquad l_0 = 10^{-33}\text{cm} \qquad m_0 = 10^{-5}\text{g} \tag{3}$$

The idea of black hole entropy had its origin in the remark that black hole formation is an irreversible process. It was found that in any process involving a black hole its surface cannot decrease; the same holds for processes involving several black holes for the sum total of their surfaces. The surface of a black hole is proportional to the square of its mass M_{bh}, and it was found that the statistical entropy S_{bh} of a Schwarzschild black hole, being a dimensionless number, has the value

$$S_{bh} = 4\pi M_{bh}^2 \qquad (4)$$

if M_{bh} is measured in the „natural" units (3). For a black hole of solar mass $M_{bh} = 2 \times 10^{33}\text{g} = 10^{38} m_0$, we get

$$S_{bh} = 10^{77} \qquad (5)$$

a large number indeed.

For a fundamental theory of black holes, star masses are as contingent as elementary particle masses; star masses are in fact defined by stability conditions which depend on atomic constants. From the point of view of pure gravitation, the „natural" limiting values of black hole entropy would be the entropy of the smallest and of the largest possible black hole. These are given by

$$M_{bh(min)} = m_0$$
$$M_{bh(max)} = M_U = M_{(\text{mass of the universe})} \qquad (6)$$

For $M_{bh(min)}$, S_{bh} is of the order of unity; for $M_{bh(max)}$, we get

$$E_4 = S_U \approx 10^{120} = E_1^3 \qquad (7)$$

If the universe is spatially compact, we may indeed consider it as something like a huge black hole in whose inside we have the good luck to live.[2] Then E_4 would be the entropy of the universe. What does such a statement mean?

Entropy is a subconcept of the general concept of information. In Boltzmann's language, the entropy of a closed system is the logarithm of the number of its possible microstates contained in its macrostate. The information content I_{mac} of knowing the macrostate alone is the actual information we possess about the system in phenomenological thermodynamics. The information content of a microstate I_{mic} is the virtual information, i.e., the information we would possess if we knew the microstate.

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[2] 0f course, there are well-known differences between a black hole and a Friedmann universe. But the interior solution of the black hole can be written as part of such a cosmos and its volumes are of the same order of magnitude.

The quantity

$$S = I_{mic} - I_{mac} \tag{8}$$

is the information on the system that we lack by only knowing the macrostate; this is what we mean by the system's entropy. S_{bh} is thus the information an outside observer loses by the process of black hole formation; it is hence the information an inside observer of the black hole might maximally gain by describing its state as it can be known to that observer observing the full microstate. Now, we are inside observers of the universe. Hence S_U would mean the information content of the universe.

3. UR THEORY

How can we define a microscopically observable information content of the universe that might be compared with this number? For this purpose we use the concepts of Weizsäcker's (1971) theory of ur alternatives, as presented in 1968; for further information on the theory cf. Weizsäcker (1971b, 1985),[3] Castell *el al.* (1975, 1977, 1979, 1981, 1983, 1985), and Drieschner (1973). Basically, the theory is a consistent interpretation of quantum theory in terms of information. In a separable Hilbert space a basis can be given by eigenvectors of a self-adjoint operator with a discrete eigenvalue spectrum. Such an operator is interpreted as an observable, i.e., as an empirically decidable, infinite-valued alternative. In fact, only finite alternatives can be empirically decided. Logically, every discrete alternative can be decided by a sequence of binary alternatives. Corresponding to that, every separable Hilbert space can be described as a tensor product of two-dimensional spaces, or as a subspace of such a tensor product. If we are permitted to describe the possible quantum states of the whole universe in a separable Hilbert space, it is mathematically trivial that the states of the universe can be built up from states of smallest, binary „subjects." These are called „urs“ by Weizsäcker (from German *Ur-Alternativen* = original alternatives).

The state space of a single ur is $\mathbf{C}^2$. It is symmetry group is $SU(2)\times U(1)$. Assuming that the theory of a composite object is invariant under the same group as that of its smallest equal parts, one concludes that any object in the universe, and the universe itself, ought to be have at least SU(2) as its symmetry group. A natural description of the quantum states of an object with a given symmetry group is given by the square integrable functions on a homogeneous space of this group. SU(2) is locally isomorphic to SO(3). Weizsäcker considered this as the reason why all physical objects

[3] A series of three shorter English papers on the content of Weizsäcker (1985) is under preparation.

can be naturally described in a three-dimensional real position space. Thus, the ur theory does not presuppose the existence of the space-time continuum; on the contrary, it *deduces* the position space from the mathematical structure of abstract quantum theory.

We can choose the parameter space of SU(2) itself as its homogeneous space. It is an $\mathbf{S}^3$. It is the simplest and thus the most natural assumption that this $\mathbf{S}^3$ *is* cosmic space. The wave function for a single ur can then be represented as a function over the $\mathbf{S}^3$ possessing only one knot line. Hence an ur is to be understood as an extremely nonlocal object. Localized states in the universe are obtained by superposition of many urs. The state space of any object has to be a subspace in the Fock space of all urs which itself is isomorphic to a subspace of the infinite tensor product of $\mathbf{C}^2$. Castell (1975) has provided a method to construct the state space of a massless particle from urs. Models for massive particles have also been found (Weizsäcker, 1985; Görnitz and Weizsäcker, 1985).

What would be the information content of the universe in this picture? The decision of one ur alternative provides one bit of information. Hence the information content of the universe may be considered as equal to the number N of urs in it. Weizsäcker (1973) gave an estimate of this number prior to the theory of black hole entropy. He subdivides the universe into elementary volumes equal to λ_n^3, where λ_n is the Compton wavelength of the nucleon. For every elementary volume a binary alternative is defined by asking whether it contains a nucleon or not. Nucleons are chosen since they form the bulk of ponderable matter, and ponderable matter is needed for actual experiments. Since the radius R of the universe is $10^{40}\,\lambda_n$, the result is

$$N = \lambda_n{}^3 = 10^{120} \qquad (9)$$

in accordance with (7).

Of course, this estimate makes use of a basis of the Hilbert space of the universe which is different from the direct counting of urs. The ur is nonlocal, while here we start from localized volumes. We assume, however, that the two bases give the same number of dimensions to the Hilbert space ($d = 2^N$), and hence the same information content.[4] In any case, the Hilbert space of this model of the universe it finite-dimensional. The universe is not Lorentz-invariant. Only in a locally tangential Minkowski space will infinite-dimensional Hilbert spaces be defined.

In order to localize a nucleon in a universe of radius R down to an accuracy comparable to λ_n, we need a superposition of $3R/\lambda_n$ urs, R/λ_n for every space dimension. Now

$$R/\lambda_n = E_3 = 10^{40} \qquad (10)$$

[4] A more detailed estimate wil be given in a subsequent paper.

This will permit us to subdivide the N urs of the universe into N/E_3 nucleons, and we get

$$E_2 = N/E_3 = 10^{80} \tag{11}$$

in accordance with (1). Generally speaking, ur theory requires the equation with

$$N = E_4 = E_2 \times E_3 \tag{12}$$

$$E_2 = N^{2/3} \qquad\qquad E_3 = N^{1/3} \tag{13}$$

The result (10) on the information content of the nucleon also has been derived from the theory of black hole entropy. For a particle of mass m falling into a black hole, the entropy rises by an amount

$$S_{diff} = 4\pi\ [(M_{bh}+m)^2 - M_{bh}^2) = 8\pi\ M_{bh} \times m$$

This is the information lost for an outside observer due to the particle falling into the black hole. The authors were surprised also to find S_{diff} to be exceedingly large. The maximum possible value of S_{diff} is reached for $M_{bh} = M_U$. In our units (3) is

$$m_n = 10^{-20} m_0 \qquad \text{and} \qquad M_U = 10^{60} m_0$$

Hence

$$S_{diff(max)} = 10^{40} \tag{14}$$

The ur theory explains this value. A nucleon "is" 10^{40} urs.

So far, our description refers only to the momentary state of the universe. Within this limit it turns out to be a trivial mathematical consequence of abstract quantum theory if we add only the postulate that the information content of the universe as presently accessible to our theory is finite. For a theory of particles and fields as well as for the history of the universe we need a law of dynamics. We leave that to later papers (cf. Görnitz, 1985) and confine ourselves to a qualitative remark. If a law of dynamics admits a noncompact symmetry group, its unitary representations need an infinite-dimensional Hilbert space. Time with an open future would mean a group of time translations in R^1. This law of dynamics will not keep the number of urs constant. This is the ur-theoretical expression of the idea that in an open future there is no upper bound to the degree of decidable alternatives; they are finite at any moment, but unbounded with proceeding time. If, in such a dynamics, we begin with a finite number $N(t_0)$ of urs, it is to be expected statistically that $N(t)$ will on the average increase monotonically with t. If, at any time, $N^{1/3} = R/\lambda_n$, and if λ_n is supposed to be the unit of length used in ponderable measuring instruments and hence defined as constant through time, this means an expansion of the universe.

These last considerations lead us back to Dirac's hypothesis, but with a so-far-undetermined function *R(t).* A more precise description of dynamics will be needed for its elaboration.

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